Mark Scheme (Results)

January 2018

Pearson Edexcel International GCSE in Further Pure Mathematics (4PM0)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Types of mark

- M marks: method marks
- A marks: accuracy marks
- B marks: unconditional accuracy marks (independent of M marks)

Abbreviations

- cao – correct answer only
- ft – follow through
- isw – ignore subsequent working
- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses A (and B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.
  If there is a choice of methods shown, then no marks should be awarded, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer on the answer line then check the working for an obvious answer.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

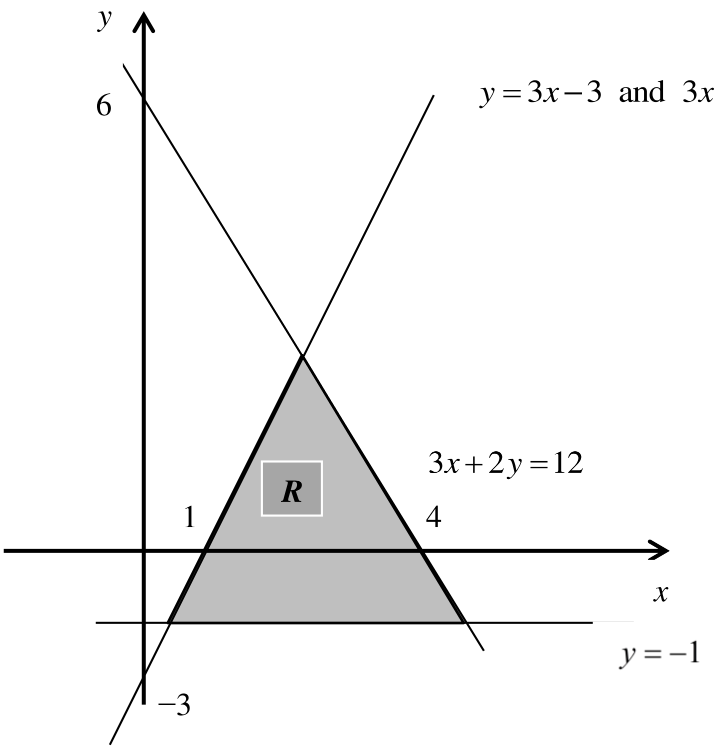
Jan 2018

4PMO Further Pure Mathematics Paper 1
Mark Scheme

Question number	Scheme	Marks
1(a)	Completes the square to find, $\mathrm{f}(x) = -2\left(x - \frac{5}{4}\right)^2 + \frac{73}{8}$ $p = -2 \quad q = -\frac{5}{4} \quad r = \frac{73}{8}$	M1 A2,1,0 (3)
	ALT $6 + 5x - 2x^2 = px^2 + 2pqx + pq^2 + r$ $\Rightarrow p = -2$ $-4q = 5 \Rightarrow q = -\frac{5}{4}$ $(-2)\left(\frac{25}{16}\right) + r = 6 \Rightarrow r = \frac{73}{8}$	M1 A1 A1 (3)
(b)	(i) $\mathrm{f}(x) = \frac{73}{8}$ (ii) $x = \frac{5}{4}$	B1ft B1ft (2)
(c) (i)	$\mathrm{g}(x) = \frac{73}{8}$	M1A1
(ii)	$x^3 - \frac{5}{4} = 0 \Rightarrow x = \sqrt[3]{\frac{5}{4}}$	B1ft (3)
		[8]

Additional Notes

Part	Mark	Guidance
(a)	M1	Takes out -2 as a common factor to give $-2\left(x\pm\frac{5}{4}\right)^2 \pm k \quad k\neq 3,\ \ k\neq 0$
	A1	For either $-2\left(x-\frac{5}{4}\right)^2 \pm k$ or $-2\left(x\pm\frac{5}{4}\right)^2+\frac{73}{8}$
	A1	For the correct values of p, q and r as shown. Accept embedded in $\mathrm{f}(x)$ $\left[\mathrm{f}(x)=\right]-2\left(x-\frac{5}{4}\right)^2+\frac{73}{8}$
	ALT	
	M1	Expands $p(x+q)^2+r$ **correctly** $\left[px^2+2pqx+pq^2+r\right]$, equates to $6+5x-2x^2$ **and** solves for at least one of p, q or r.
	A1	For **two** correct of p, q or r.
	A1	For all three correct.
(b)	B1ft	For $\mathrm{f}(x)='\frac{73}{8}'$ oe. (9.125) follow through their $\frac{73}{8}$ (unless they use calculus in which case if $\mathrm{f}(x)=\frac{73}{8}$ is correct here then award the mark.
	B1ft	For $x='\frac{5}{4}'$ follow through their $\frac{5}{4}$
ALT		Uses calculus; it must be clear which are the values of $\mathrm{f}(x)$ and which of x.
(c)	M1	For $-2\left(x^3\pm'\frac{5}{4}'\right)^2\pm'\frac{73}{8}'\Rightarrow \mathrm{g}(x)='\frac{73}{8}'$ follow through their $\frac{73}{8}$ for this mark. The above need not be seen. Adequate evidence for this mark is $\mathrm{g}(x)='\frac{73}{8}'$
	A1	For $\mathrm{g}(x)=\frac{73}{8}$
	B1ft	For $x=\sqrt[3]{'\frac{5}{4}'}$ oe e.g. $\left(\frac{15}{12}\right)^{\frac{1}{3}}$ ft $\frac{5}{4}$ Do not accept 1.07721 mark.
ALT		Uses calculus
(c)	M1	$\frac{\mathrm{d}y}{\mathrm{d}x}=15x^2-12x^5=0\Rightarrow x=\sqrt[3]{\frac{5}{4}}\Rightarrow \mathrm{g}(x)=6+5\left(\sqrt[3]{'\frac{5}{4}'}\right)^3-2\left(\sqrt[3]{'\frac{5}{4}'}\right)^6='\frac{73}{8}'$
	A1	For $\mathrm{g}(x)=\frac{73}{8}$ oe (9.125)
	B1ft	For $x=\sqrt[3]{'\frac{5}{4}'}$ oe e.g. $\left(\frac{15}{12}\right)^{\frac{1}{3}}$ ft $\sqrt[3]{\frac{5}{4}}$ from their differentiation. Do not accept 1.07721... for this mark.
Note: If answers to (b) and (c) are not labelled (i) or (ii) at least one of their values must be labelled correctly.		

Question number	Scheme	Marks
2 (a)	 $y=3x-3$ and $3x+2y=12$	B1 B1 (2)
(b)	Correct line drawn $y=-1$ Correct region shaded	B1 B1 (2)
(c)	see table below	M1A1 M1A1 (4) [8]

Vertex	$(2,3)$	$\left(\frac{14}{3},-1\right)$	$(\frac{2}{3},-1)$
$P=4x-y$	5	$\frac{59}{3}$	$\frac{11}{3}$
		greatest	

Additional Notes		
Part	**Mark**	**Guidance**
(a)	B1	Either $y = 3x - 3$ **or** $3x + 2y = 12$ drawn correctly Intersections on axes of $y = 3x - 3$ are (0, —3) and (1, 0) Intersections on axes of $3x + 2y = 12$ are (4, 0) and (0, 6)
	B1	Both $y = 3x - 3$ **and** $3x + 2y = 12$ drawn correctly.
(b)	B1	Line y = —1 drawn correctly and marked. This line can be implied by the shading.
	B1	Correct region shaded in or out. *R* need not be explicitly labelled
(c)	M1	For attempting to find correct coordinates of at least one intersection with the line y = —1. i.e. either $\left(\frac{14}{3}, -1\right)$ or $\left(\frac{2}{3}, -1\right)$. Accept 4.6, 4.7, 4.8 or 0.6, 0.7, 0.8 (from their graph) for this mark.
	A1	**This is an M mark in Epen**. For $\left(\frac{14}{3}, -1\right)$ Accept 4.6, 4.7, 4.8 for $\frac{14}{3}$
	M1	For substituting their $\left(\frac{14}{3}, -1\right)$ into *P*. Allow 4.6, 4.7 or 4.8 (from their graph) for this mark.
	A1	For $P = \frac{59}{3}$ Accept awrt 19.7
	ALT	
	M1	Slope of objective function line is 4 Identifies the intersection of $3x + 2y = 12$ and $y = -1$ as the point where *P* is greatest and attempts to find the point of intersection by
	A1	**This is an M mark in Epen**. For finding $\left(\frac{14}{3}, -1\right)$ Accept 4.6, 4.7, 4.8 for $\frac{14}{3}$
	M1	For substituting their $\left(\frac{14}{3}, -1\right)$ into *P*. Allow 4.6, 4.7 or 4.8 (from their graph) for this mark.
	A1	For $P = \frac{59}{3}$ Accept awrt 19.7

Question number	Scheme	Marks
3	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}=27\right)$ $r=\frac{3h}{2}$ $V=\frac{1}{3}\pi r^2 h \Rightarrow V=\frac{3}{4}\pi h^3$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{9}{4}\pi h^2$ $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}h}{\mathrm{d}V}$ $\frac{\mathrm{d}h}{\mathrm{d}t}=27\times\frac{4}{9\pi h^2}=27\times\frac{4}{9\pi 4^2}=0.23873\ldots\ \frac{\mathrm{d}h}{\mathrm{d}t}=0.239$	 B1 M1A1 M1 M1dd A1 [6]

Additional Notes	
Mark	**Guidance**
B1	For using the given $r = 1.5h$ to find the **correct** expression for the volume in terms of h only. Need not be simplified. Accept $V = \frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 h$ or $V = \frac{1}{3}\pi \times \frac{9h^2}{4} \times h$ **sc** You may see $27 = \frac{3}{4}\pi h^3$ Award B1 here if this is later differentiated and used correctly.
M1	For attempting to differentiate their V provided it is in terms if h only. Must be a dimensionally correct V. See general guidance for the definition of an attempt.
A1	For the correct derivative $\frac{dV}{dh} = \frac{9}{4}\pi h^2$
M1	For a correct expression of chain rule. Accept any correct equivalent. Eg., $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ oe Please check this carefully. Chain rule may not be explicitly stated, but may be implied from correct work.
M1dd	For substituting $h = 4$ and $\frac{dV}{dt} = 27$ into their expression of chain rule. It must be correct, but not necessarily with $\frac{dh}{dt}$ as the subject Note: this mark is dependent on **BOTH** previous Method marks scored.
A1	For $\frac{dh}{dt} = 0.239$ rounded correctly.
ALT	
B1	For using the given $r = 1.5h$ to find the **correct** expression for the volume in terms of h only.
M1	For attempting to differentiate their V wrt to t provided V is in terms if h only. Must be a dimensionally correct V. $\frac{dV}{dt} = \frac{9}{4}\pi h^2 \frac{dh}{dt}$
A1	For a correct expression for $\frac{dV}{dt}$ in terms of h and $\frac{dh}{dt}$
M1	For re-arranging their $\frac{dV}{dt} = \frac{9}{4}\pi h^2 \frac{dh}{dt}$ to $\frac{dh}{dt} = \frac{4}{9\pi h^2} \times \frac{dV}{dt}$ Please check their re-arrangement, it must be correct for this mark.
M1dd	For substituting $h = 4$ and $\frac{dV}{dt} = 27$ into their $\frac{dh}{dt}$ **Note: This M mark and the previous M mark may be in either order.**
A1	For $\frac{dh}{dt} = 0.239$ rounded correctly.

Question number	Scheme	Marks
4 (a)	When P is at rest $v = 0$ $2t^2 - 16t + 30 = 0 \Rightarrow (2t-6)(t-5) = 0$ $t = 3, 5$	M1A1 (2)
(b)	$\frac{dv}{dt} = 4t - 16$ $t = 3 \quad \frac{dv}{dt} = -4$ $t = 5 \quad \frac{dv}{dt} = 4$	M1 M1 A1 (3)
(c)	$s = \int (2t^2 - 16t + 30)\, dt = \frac{2t^3}{3} - 8t^2 + 30t \ (+c)$ when $t = 0$, $s = -4 \Rightarrow c = -4$ $s = \frac{2 \times 3^3}{3} - 8 \times 3^2 + 30 \times 3 - 4 = 32$ (m)	M1 B1 A1 (3) [8]

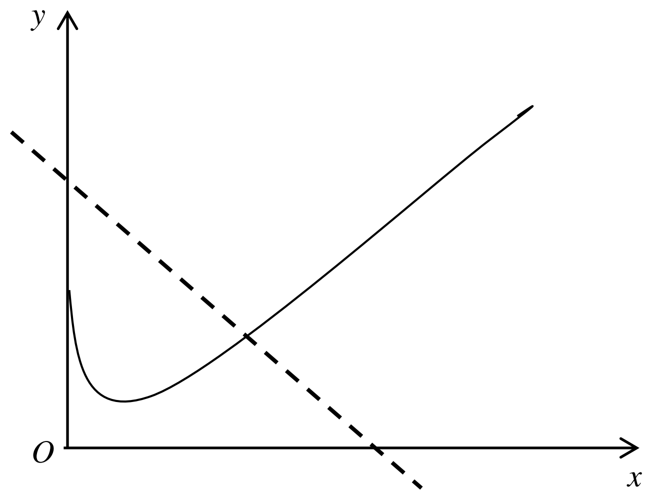
Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	Sets $2t^2 - 16t + 30 = 0$ **and** attempts to solve the quadratic. (See General Guidance for the definition of an attempt) They must achieve two values of *t* for this mark
	A1	For $t = 3, 5$ Accept $t = 3, 5$ without working shown.
(b)	M1	For an attempt to differentiate the **given** *v* (See General Guidance for the definition of an attempt)
	M1	For substituting **both** values of *t* to achieve **two** values for the acceleration.
	A1	$\frac{dv}{dt} = -4$ and 4
(c)	M1	For an attempt to integrate the **given** *v* **and** substitute *t* = 3 into their integrated expression and find a value for *s*. (See general guidance for the definition of an attempt) *c* is not required for this mark ALT using definite integration; Integrated and evaluated $\left['\frac{2t^3}{3} - 8t^2 + 30t' \right]_0^3 (-4)$ This must be a complete method for this mark.
	B1	Uses the information given to find that $c = -4$ ALT using definite integration; subtracts 4 from their evaluated integrated expression.
	A1	For *s* = 32 (m) cso

Question number	Scheme	Marks
5 (a)		

x	0	0.5	1	1.5	2	3	4
y	**2(.00)**	1.42	**1.5(0)**	2.15	**3.33**	7.25	**13.2(0)**

B1B1
(2)

(b)



B1ftB1ft
(2)

(c)

$$\frac{x^3+2}{x+1}=ax+b$$

$$x^3+2=(x+1)(ax+b)=ax^2+x(a+b)+b$$

$$x^3-ax^2-x(a+b)+b-2\equiv x^3+x^2-3x-2$$

$\Rightarrow a=-1$ and $b=4$ so required line is $y=-x+4$ — M1A1A1

ALT

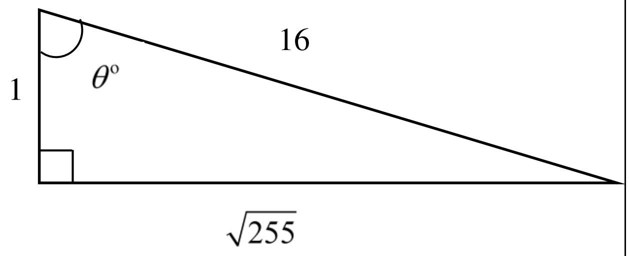
$$x^3+x^2-3x-2=0\Rightarrow x^3+2=-x^2+3x+4\Rightarrow x^3+2=(x+1)(4-x)$$

$\Rightarrow 4-x=\dfrac{x^3+2}{x+1}\Rightarrow$ line required is $y=4-x$ — {M1A1A1}

Line $y=4-x$ drawn $\Rightarrow x=1.6$ — M1A1

(5)
[9]

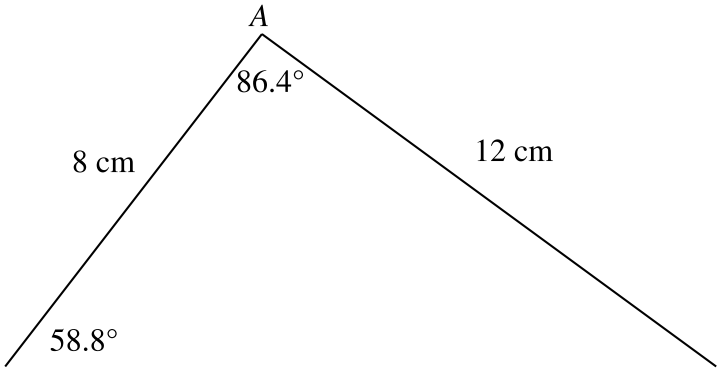
Additional Notes		
Part	**Mark**	**Guidance**
(a)	B1	For any two correct values (correctly rounded) Accept 2.00 and 1.50 and also $3.3\dot{3}$ or $3.\dot{3}$
	B1	For all four correct values (correctly rounded) Accept 2.00 and 1.50
(b)	B1ft	For their points all correctly plotted within half of one square. Ignore a missing point provided the line goes through the correct point.
	B1ft	For **all** of their points joined in a smooth curve.
(c)	M1	Equates $\frac{x^3+2}{x+1}=ax+b$, and multiplies out **correctly**
	A1	For either $a=-1$ **or** $b=4$
	A1	For $a=-1$ **and** $b=4$. For the correct line stated, $y=-x+4$ oe seen, award A1A1 We do not need to see $y=4-x$ stated explicitly.
	M1	For **their** $y=-x+4$ drawn correctly Intersections with coord axes (4,0) (0,4) For the correct line $y=-x+4$ drawn award M1A1A1M1
	A1	For x = 1.6 only
	ALT	
	M1	Rearranges $x^3+x^2-3x-2=0$ into the numerator of x^3+2 on one side and $-x^2+3x+4$ on the other. Must be correct
	A1	Attempts to factorise the quadratic (See general guidance for the definition of an attempt)
	A1	A correct re-arrangement $4-x=\frac{x^3+2}{x+1}$ with the line $y=4-x$ oe seen This mark can be implied from a correct line drawn.
	M1	For **their** $y=-x+4$ drawn correctly Intersections with coord axes (4,0) (0,4) For the correct line $y=-x+4$ drawn award M1A1A1M1
	A1	For x = 1.6 only
Note: Do not accept x = 1.6 seen without the correct line y = 4 — x		

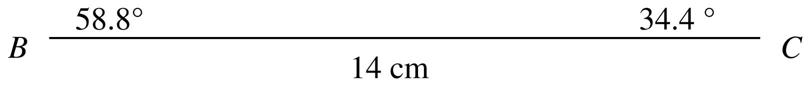
Question number	Scheme	Marks
6 (a)	$\tan\theta^{\circ} = \sqrt{255}$ $1^2 + 255 = 256$ $\sqrt{256} = 16$  $\Rightarrow \cos\theta^{\circ} = \dfrac{1}{16}$ *	M1A1cso (2)
(b)	$\cos\theta^{\circ} = \dfrac{1}{16} = \dfrac{x^2 + (x+4)^2 - (2x-2)^2}{2 \times x \times (x+4)}$ $\Rightarrow 0 = 17x^2 - 124x - 96$ $\Rightarrow x = \dfrac{124 \pm \sqrt{124^2 - 4 \times 17 \times (-96)}}{2 \times 17} = 8$ (other root not needed)	M1A1A1 M1A1 (5)
(c)	**Method 1** $\{AB = 8, \quad AC = 12, \quad BC = 14\}$ Uses sine rule to find ABC $\left[\theta^{\circ} = \tan^{-1}\sqrt{255} = 86.416...\right]$ $\dfrac{\sin 86.416}{14} = \dfrac{\sin ABC}{12} \Rightarrow \angle ABC = \sin^{-1} 0.855467... = 58.8^{\circ}$	M1A1 (2)
	Method 2 $\{AB = 8, \quad AC = 12, \quad BC = 14\}$ Uses cosine rule $\cos ABC = \dfrac{8^2 + 14^2 - 12^2}{2 \times 8 \times 14} = 0.5178... \Rightarrow ABC = 58.8^{\circ}$	{M1A1} {(2)}
(d)	Area = $\dfrac{1}{2} \times 8 \times 14 \times \sin 58.8 = 47.9$ (cm^2) **ALT** Uses Heron's formula $s = \dfrac{8 + 12 + 14}{2} = 17$ $A = \sqrt{17(17-8)(17-12)(17-14)} = 47.9$	M1A1 (2) {M1A1}

		[11]

Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	Uses the tangent ratio with Pythagoras theorem to establish that the hypotenuse is '16' **.** This is a show question, we **must see** evidence of Pythagoras theorem used for this mark.
	A1	For $\cos\theta^{\circ} = \frac{1}{16}$ cso
	ALT	
	M1	$\tan\theta = \sqrt{255} \Rightarrow 255 = \frac{\sin^2\theta}{\cos^2\theta} \Rightarrow \sin^2\theta = 255\cos^2\theta$ $\cos^2\theta + 255\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{256}$ There must be a complete method for the award of this mark.
	A1	For $\cos\theta^{\circ} = \frac{1}{16}$ cso
(b)	M1	For attempting to use cosine rule. (Any attempt to use sine rule is M0)
	A1	Uses a correct cosine rule either form, substitutes $\cos\theta^{\circ} = \frac{1}{16}$ Alternative form of cosine rule: $(2x-2)^2 = (x+4)^2 + x^2 - 2\times(x+4)\times x \times \frac{1}{16}$ (Allow $\cos 86.4^{\circ}$ for this mark)
	A1	For forming a correct 3TQ
	M1	Attempts to solve **their** 3TQ (See general guidance)
	A1	$x = 8$ (ignore other root)
(c)	M1	Uses correct trigonometry (sine or cosine rule using their value for x) and achieves a value for angle ABC.
	A1	$\angle ABC = 58.8^{\circ}$
(d)	M1	Uses $\frac{1}{2}ab\sin C$ correctly with their value of x and their angle ABC (if they use that angle) to find the area of the triangle.
	A1	For 47.9 (cm^2)
	ALT	
	M1	Uses a correct Heron's formula with values derived from their x.
	A1	For 47.9 (cm^2)

Useful sketch





Question number	Scheme	Marks
7 (a)	$\left(1-4x^2\right)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\right)\left(-4x^2\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-4x^2\right)^2}{2!}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-4x^2\right)^3}{3!}$ $\left(1-4x^2\right)^{-\frac{1}{2}} = 1+2x^2+6x^4+20x^6+\ldots$	M1A1A1 (3)
(b)	$-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x \rvert < \frac{1}{2}$	B1 (1)
(c)	$\left(3+x\right)\left(1+2x^2+6x^4\right) = 3+x+6x^2+2x^3+18x^4$	M1M1A1 (3)
(d)	$\int_0^{0.3} \frac{3+x}{\sqrt{\left(1-4x^2\right)}}\,\mathrm{d}x = \left[3x+\frac{x^2}{2}+2x^3+\frac{x^4}{2}+\frac{18x^5}{5}\right]_0^{0.3} = 1.011798 \approx 1.01$ (3sf)	M1A1M1d A1 (4) [11]

Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	For an attempt at the binomial expansion. Minimally acceptable attempt: • The first term of the expansion must be 1 • The power of x must be correct in each term **Note:** ($-4x^2$ must be used correctly at least once). • The denominators must be correct
	A1	First term of 1 and at least one term in x correctly simplified
	A1	The complete expansion completely correct as shown.
(b)	B1	For either form of the validity $-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x \rvert < \frac{1}{2}$
(c)	M1	Shows an intention to multiply their expansion by $(3+x)$
	M1	Multiplies out **their** expansion by $(3+x)$ to achieve at least five terms starting with the first term = 3, in ascending powers of x up to x^4 (need not be in order of ascending powers of x for this mark)
	A1	For a fully correct expansion $3+x+6x^2+2x^3+18x^4$ These terms need not be in order
(d)	M1	For attempting to integrate **their** expansion which must have a minimum of 5 terms up to x^4
	A1	For a fully correct integrated expression $3x+\frac{x^2}{2}+2x^3+\frac{x^4}{2}+\frac{18x^5}{5}$
	M1d	Substitutes 0.3 (and 0) into their integrated expansion
	A1	For 1.01 correctly rounded

Question number	Scheme	Marks
8(a)	$\dfrac{ar^5}{ar}=4\Rightarrow r^4=4\Rightarrow r=\pm\sqrt{2}$	M1A1 (2)
(b)	$ar^2+ar^6=30\Rightarrow a\left(r^2+r^6\right)=30$ $a\left[\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^6\right]=30\Rightarrow 10a=30\Rightarrow a=3$	M1A1A1 (3)
(c)	$S_{10}=\dfrac{3\left(\left(\sqrt{2}\right)^{10}-1\right)}{\sqrt{2}-1}=\left\{\dfrac{93}{\sqrt{2}-1}\right\}$ or awrt 224.5 or $93\left(\sqrt{2}+1\right)$	M1A1 (2)
(d)	$2400<3\times\left(\sqrt{2}\right)^{(n-1)}\Rightarrow\left(\sqrt{2}\right)^{(n-1)}>800$ $n-1>\log_{\sqrt{2}}800\Rightarrow n-1>19.287...\Rightarrow n>20.287...$ $n=21$	M1 M1dA1 (3) [10]

Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	Attempts to find r by solving the equation $ar^5 = 4ar$ (oe) to achieve two values of r
	A1	For $r = \pm\sqrt{2}$ $\pm$ required for this mark. Accept $\pm\sqrt[4]{4}$ for this mark
(b)	M1	Uses $ar^2 + ar^6 = 30$ and substitutes their r to form an equation to find a.
	A1	For a correct equation $a\left[\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^6\right] = 30$ oe $\left\{a\left[\left(\sqrt[4]{4}\right)^2 + \left(\sqrt[4]{4}\right)^6\right] = 30\right\}$
	A1	For $a = 3$
(c)	M1	Uses a correct summation formula with their r and their a Accept adding up $a + ar + ar^2 + ... + ar^9$ provided no terms are missing. There must be 10 terms.
	A1	For awrt to 225 or the exact answer $93\left(\sqrt{2}+1\right)$ oe e.g. $\dfrac{-93}{1-\sqrt{2}}$
(d)	M1	Sets up an inequality (either < or > for this mark – accept =) using their values with a correct formula for U_n $\left[2400 < '3' \times \left('\sqrt{2}'\right)^{n-1}\right]$ Ft their a and r for this mark.
	M1d	Attempts to solve their equation using logarithms correctly. Using logs base 10 (or e or $\sqrt{2}$ or $\sqrt[4]{4}$) $n - 1 > \dfrac{\lg 800}{\lg\sqrt{2}}$ Ft their a and r for this mark.They must reach a value for n **for the award of this mark.**
	A1	$n = 21$ For the award of this mark, the correct inequality must have been used at least once .
	ALT (trial and improvement)	
	M1	Uses a correct formula for U_n and attempt to evaluate either n = 20 **or** n = 21 $\{t_{20}$=2172.23... $t_{21} = 3072\}$
	M1d	Uses a correct formula for an attempt to evaluate both n = 20 **and** n = 21
	A1	For n = 21

Question number	Scheme	Marks
9 (a)	$x^2 - \text{sum} \times x + \text{product} = 0$ $x^2 + \frac{5}{2}x - 5 = 0$ $2x^2 + 5x - 10 = 0$ or integer multiples	M1A1 (2)
(b) (i)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(\frac{25}{4}\right) + 10 = \frac{65}{4}$	M1A1
(ii)	$(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ $\Rightarrow \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -\frac{125}{8} + 15\left(-\frac{5}{2}\right) = -\frac{425}{8}$ **ALT** $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2) = \left(-\frac{5}{2}\right)\left(\frac{73}{4} + 5\right) = -\frac{425}{8}$	M1A1A1 (5) {M1A1A1}
(c)	Product $\left(\alpha - \frac{1}{\alpha^2}\right) \times \left(\beta - \frac{1}{\beta^2}\right) = \left(\frac{\alpha^3 - 1}{\alpha^2}\right)\left(\frac{\beta^3 - 1}{\beta^2}\right) = \frac{\alpha^3\beta^3 - (\alpha^3 + \beta^3) + 1}{\alpha^2\beta^2}$ $= \frac{-125 - -\frac{425}{8} + 1}{36} = -\frac{567}{200}$ Sum $\left(\alpha - \frac{1}{\alpha^2}\right) + \left(\beta - \frac{1}{\beta^2}\right) = \left(\frac{\alpha^3 - 1}{\alpha^2}\right) + \left(\frac{\beta^3 - 1}{\beta^2}\right)$ $= \frac{\alpha^3\beta^2 - \beta^2 + \alpha^2\beta^3 - \alpha^2}{\alpha^2\beta^2} = \frac{\alpha^2\beta^2(\alpha+\beta) - (\alpha^2 + \beta^2)}{\alpha^2\beta^2}$ $= \frac{25\left(-\frac{5}{2}\right) - \frac{65}{4}}{25} = -\frac{63}{20}$ oe Equation	M1 A1 M1 A1

	Sum = $-\frac{63}{20}$, Product = $-\frac{567}{200}$ $\Rightarrow x^2+\frac{63}{20}x-\frac{567}{200}\ (=0)$ $x^2+\frac{314}{100}x-\frac{567}{200}\ (=0)$ M1 $200x^2+630x-567=0$ A1	M1A1 (6) [13]

Additional Notes			
Part	**M**	**Guidance**	
(a)	M1	Forms a quadratic equation with the given product and sum $\left(x^2+\frac{5}{2}x-5\right)$ = 0 not required for this mark. Allow y =... for this mark	
	A1	For $2x^2+5x-10=0$ or equivalent equation with integer coefficients only. Look out for = 0 which must be present.	
(b) (i)	M1	Uses the **correct** algebra to form $\alpha^2+\beta^2$ **and** substitutes the given values of the sum and product.	
	A1	For $\alpha^2+\beta^2=\frac{65}{4}$	
(ii)	M1	Uses the **correct** algebra to form an expression for $\alpha^3+\beta^3$ For example; $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ or $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$	Their algebraic expansion must be sufficiently arranged to allow substitution of $\alpha^2+\beta^2$, $\alpha+\beta$ and $\alpha\beta$
	A1	Substitutes the given values for the sum and product into their form of $\alpha^3+\beta^3$	
	A1	For $\alpha^3+\beta^3=-\frac{425}{8}$ oe	
(c)	M1	**Product** For the **correct** algebra to achieve $\frac{\alpha^3\beta^3-(\alpha^3+\beta^3)+1}{\alpha^2\beta^2}$ or $\alpha\beta-\left(\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}\right)+\frac{1}{\alpha^2\beta^2}$ and substitutes their $(\text{product})^3$, $(\text{product})^2$ and their $\alpha^3+\beta^3$ Their algebra must be sufficient to substitute $\alpha\beta$, $\alpha^3+\beta^3$ and $\alpha^2\beta^2$ in directly.	
	A1	Product $=-\frac{567}{200}$ oe	
	M1	**Sum** For the **correct** algebra to achieve $\frac{\alpha^2\beta^2(\alpha+\beta)-(\alpha^2+\beta^2)}{\alpha^2\beta^2}$ or $\alpha+\beta-\left(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}\right)$ (but in a from where the sum and product can be substituted) and substitutes their $(\text{product})^2$ and their $\alpha^2+\beta^2$	

	A1	$\text{Sum} = -\frac{63}{20}$
	M1	**Equation** Uses their product and their sum correctly to form a quadratic equation $x^2 + '\frac{63}{20}'x - '\frac{567}{200}'$ (= 0 not required for this mark) Check their signs are correct Allow $y =$... for this mark
	A1	For the correct equation as shown with integer coefficients . Accept equivalent integer values eg., $2000x^2 + 6300x - 5670 = 0$

Question number	Scheme	Marks
10 (a)	$\cos 2\theta = \cos^2\theta - \sin^2\theta \Rightarrow \cos 2\theta = \cos^2\theta - \left(1-\cos^2\theta\right)$	M1M1
	$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1}{2}\left(\cos 2\theta + 1\right)$ *	A1cso (3)
(b)	(Uses $\cos^2\theta + \sin^2\theta = 1$ to give) $\cos 2\theta = 1 - 2\sin^2\theta$ seen anywhere	B1
	$4\cos^4\theta = \cos^2 2\theta + 2\cos 2\theta + 1 \Rightarrow$ $4\cos^4\theta = \frac{1}{2}\left(\cos 4\theta + 1\right) + 2\left(1 - 2\sin^2\theta\right) + 1 \Rightarrow$	M1 M1
	$8\cos^4\theta = \cos 4\theta + 1 + 4 - 8\sin^2\theta + 2 \Rightarrow$ $\cos 4\theta = 8\cos^4\theta + 8\sin^2\theta - 7$ *	M1 A1cso (5)
(c)	$16\cos^4\left(\theta - \frac{\pi}{6}\right) + 16\sin^2\left(\theta - \frac{\pi}{6}\right) - 15 = 0$ $\Rightarrow 8\cos^4\left(\theta - \frac{\pi}{6}\right) + 8\sin^2\left(\theta - \frac{\pi}{6}\right) - 7 = \frac{1}{2}$	M1A1
	$\cos 4\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2} \Rightarrow 4\left(\theta - \frac{\pi}{6}\right) = \pm\frac{\pi}{3}, \pm\frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{12}$ or decimal equivalents awrt 0.79, 0.26	M1A1 (4)
(d)	$\int_0^{\frac{\pi}{2}}\left(8\cos^4\theta + 8\sin^2\theta + 2\sin 2\theta\right)\,\mathrm{d}\theta = \int_0^{\frac{\pi}{2}}\left(\cos 4\theta + 2\sin 2\theta + 7\right)\,\mathrm{d}\theta$	
	$\Rightarrow = \left[\frac{\sin 4\theta}{4} - \cos 2\theta + 7\theta\right]_0^{\frac{\pi}{2}} = \left[\left(0 - (-1) + \frac{7\pi}{2}\right) - (0 - 1 + 0)\right] = 2 + \frac{7}{2}\pi$	M1M1M1dd A1 (4)
		[16]

Additional Notes		
Part	**Mark**	**Guidance**
(a)	M1	Uses the given identity to write $\cos 2\theta = \cos^2\theta - \sin^2\theta$
	M1	Uses the identity $\cos^2 A + \sin^2 A = 1$ to form an identity in $\cos 2\theta,\ \cos^2\theta$ and 1 only
	A1	For the correct identity as shown.
(b)	**Way 1**	
	B1	Uses $\cos^2\theta + \sin^2\theta = 1$ to give $2\sin^2\theta = 1 - \cos 2\theta$ (seen anywhere) Or uses the identity $\cos^2 A + \sin^2 A = 1$ to replace $\sin^2\theta$
	The following is a general guide for marking this part. You may see the method in a different order.	
	M1	For expanding $\cos^4\theta = \left[\frac{1}{2}(\cos 2\theta + 1)\right]^2 \Rightarrow \frac{1}{4}(\cos^2 2\theta + 2\cos 2\theta + 1)$ The expansion for $(\cos 2\theta + 1)^2$ must be correct for this mark Look for $2\cos^2 2\theta + 4\cos 2\theta + 2$
	M1d	For substituting $\frac{1}{2}(\cos 4\theta + 1)$ into $\cos^2 2\theta$
	M1d	Eliminates $\cos 2\theta$ to leave only $\cos 4\theta \pm k$
	A1	For the correct $\cos 4\theta = 8\cos^4\theta + 8\sin^2\theta - 7$ cso This is a show question. There must be no errors in this proof.
	Way 2	
	B1	Uses $\cos^2\theta + \sin^2\theta = 1$ to give $2\sin^2\theta = 1 - \cos 2\theta$ (seen anywhere) Or uses the identity $\cos^2 A + \sin^2 A = 1$ to replace $\sin^2\theta$
	M1	For expanding $\cos^4\theta = \left[\frac{1}{2}(\cos 2\theta + 1)\right]^2 \Rightarrow \frac{1}{4}(\cos^2 2\theta + 2\cos 2\theta + 1)$ The expansion for $(\cos 2\theta + 1)^2$ must be correct for this mark Look for $2\cos^2 2\theta + 4\cos 2\theta + 2$
	M1d	For substituting $\frac{1}{2}(\cos 4\theta + 1)$ into $\cos^2 2\theta$
	M1d	Eliminates $\cos^2\theta$ to leave only $\cos 4\theta \pm k$
	A1	For the correct $\cos 4\theta = 8\cos^4\theta + 8\sin^2\theta - 7$ cso

		This is a show question. There must be no errors in this proof.
	Way 3	This is the general method for marking this part starting with $\cos 4\theta$
	B1	For using either $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ or $\cos 4\theta = 2\cos^2 2\theta - 1$
	M1	For expanding; $\left(\cos^2\theta - \sin^2\theta\right)^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta$ correctly
	M1d	For expanding; $\sin^4\theta = 1 - 2\cos^2\theta + \cos^4\theta$ or $\sin^4\theta = -1 + 2\sin^2\theta + \cos^4\theta$ To be in terms of $\cos^4\theta$ and $\sin^2\theta$ or $\cos^2\theta$ This must be correct for this mark
	M1d	For eliminating all $\cos^2\theta$ by using $1 - \sin^2\theta$ Ignore incorrect integer values for this mark
	A1	For the correct final answer as shown: $\cos 4\theta = 8\cos^4\theta + 8\sin^2\theta - 7$
(c)	M1	For the correct equation $8\cos^4\left(\theta - \frac{\pi}{6}\right) + 8\sin^2\left(\theta - \frac{\pi}{6}\right) - 7 = \frac{1}{2}$ **and** subsequent substitution to give $\cos 4\left(\theta - \frac{\pi}{6}\right) = k$ where $-1 < k < 1$ $k \neq 0$ For this mark accept a substitution for $\theta - \frac{\pi}{6}$ or $4\left(\theta - \frac{\pi}{6}\right)$
	A1	$\cos 4\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2}$ For this mark accept a substitution for $\theta - \frac{\pi}{6}$ or $4\left(\theta - \frac{\pi}{6}\right)$
	M1	For $\cos 4\left(\theta - \frac{\pi}{6}\right) = '\frac{1}{2}' \Rightarrow \left(\theta - \frac{\pi}{6}\right) = \pm'\frac{\pi}{12}' \Rightarrow \theta = '\frac{\pi}{4}', '\frac{\pi}{12}'$ (At least one value for θ)
	A1	$\theta = \frac{\pi}{4}$, $\frac{\pi}{12}$ or awrt 0.79, 0.26 Ignore any extra values outside of the range. Penalise extra values within range by deducting the A mark.
	ALT	
	M1	Uses the identity $\cos^2 A + \sin^2 A = 1$ to replace $\cos^2\theta$ and forms a 3TQ in $\cos^2\theta$ to give as a minimum $16\cos^4\left(\theta - \frac{\pi}{6}\right) - 16\cos^2\left(\theta - \frac{\pi}{6}\right) \pm k = 0$
	A1	For the correct 3TQ $16\cos^4\left(\theta - \frac{\pi}{6}\right) - 16\cos^2\left(\theta - \frac{\pi}{6}\right) + 1 = 0$
	M1	Solves 3TQ to give $\cos\left(\theta - \frac{\pi}{6}\right) = \pm 0.96591$ and/or ± 0.25865 Accept just positive values
	A1	$\theta = \frac{\pi}{4}$, $\frac{\pi}{12}$ or awrt 0.79, 0.26

		Ignore any extra values outside of the range. Penalise extra values within range by deducting the A mark.
(d)	M1	Replaces $8\cos^4\theta+8\sin^2\theta+2\sin 2\theta$ by $\cos 4\theta+2\sin 2\theta+7$
	M1	Integrates their expression, provided it does not contain any powers of cos or sin. Minimally acceptable integration is $\int k\cos 4\theta \Rightarrow \frac{\pm k\sin 4\theta}{4}$ **or** $\int k\sin 2\theta \Rightarrow \frac{\pm k\sin 2\theta}{2}$
	M1dd	For substituting $\frac{\pi}{2}$ **and** 0 into **their** integrated expression
	A1	For $2+\frac{7}{2}\pi$ oe